MEMORANDUM PROBABILITY GR 11 & 12

Worksheet 11 A

1. $P(A \text{ or } C) = P(A) + P(C) - P(A \cap C)$ $[P(A \cap C) = 0]$
$0,43 = 0,24 + P(C) - 0$
$P(C) = 0,43 - 0,24 = 0,19$

2.1 $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$
$0,52 = 0,36 + 0,24 - P(A \cap B)$
$P(A \cap B) = 0,60 - 0,52$
$= 0,08$

2.2 $P(A') = 1 - P(A)$
$= 1 - 0,36 = 0,64$

3.1 $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$
$0,75 = 0,6 + 0,35 - P(A \cap B)$
$P(A \cap B) = 0,95 - 0,75 = 0,2$

3.2 $P(A) \times P(B) = 0,6 \times 0,35 = 0,21$
$[P(A \cap B) = 0,2]$

$\therefore P(A) \times P(B) \neq P(A \cap B)$
$\therefore$ A and B are not independant

3.3 A and B are not mutually exclusive, because $P(A \cap B) \neq 0$

3.4 Complementary events must be mutually exclusive.
$P(A \cap B) \neq 0$

4. $P(K \text{ or } L) = 0,74$
$P(K) = 0,45$
$P(L) = x$

4.1 Mutually exclusive: $P(K \cap L) = 0$

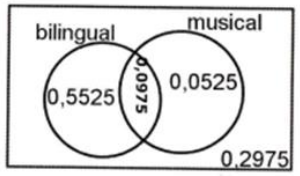
$\therefore P(K \cup L) = P(K) + P(L) - P(K \cap L)$
$0,74 = 0,45 + x - 0$
$0,29 = x$

4.2 Independent: $P(K) \times P(L) = P(K \cap L)$

$P(K \cup L) = P(K) + P(L) - P(K \cap L)$
$0,74 = 0,45 + x - (0,45 \times x)$
$0,29 = 0,55x$
$\frac{29}{55} = x$

Worksheet 11 A

5.1



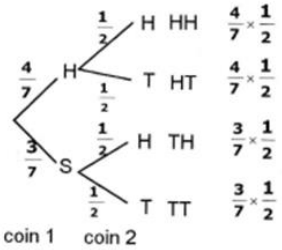
Bilingual (0,65) Musical (0,15)
P(Musical and Bilingual)
$= P(\text{Musical}) \times P(\text{Bilingual})$
$= 0,15 \times 0,65$
$= 0,0975$ [if independent]

5.2 $P(M \cup T) = P(M) + P(T) - P(M \cap T)$
$= 0,15 + 0,65 - 0,0975$
$= 0,7025$
$1 - P(M \cup T) = 1 - 0,7025 = 0,2975$

5.3 P(only one talent)
$= 0,5525 + 0,0525 = 0,605$
[Venn-diagram will help.]

Worksheet 11 B

1.1

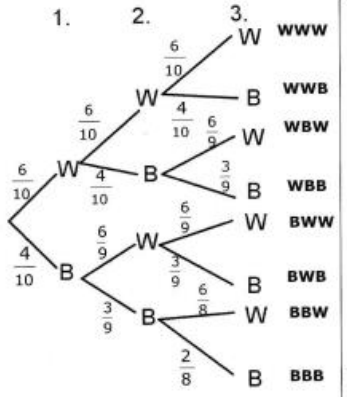


1.2 Independent – casting of coin 1 has no influence on the outcome of coin 2.

1.3 $P(HH) = \frac{4}{7} \times \frac{1}{2} = \frac{4}{14}$
$= \frac{2}{7}$ (0,2857... ≈ 0,29)

Worksheet 11 B

2.1



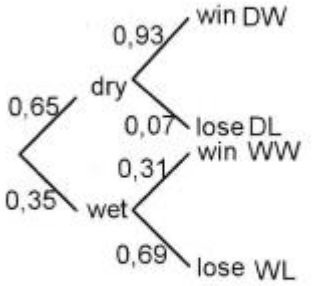
2.2.1 $P(BBB) = \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} = \frac{1}{30}$
$= 0,03\dot{3} \approx 0,03$

2.2.2 P(2 blue marbles)
$= P(WBB) + P(BWB) + P(BBW)$
$= \left(\frac{6}{10} \times \frac{4}{10} \times \frac{3}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9} \times \frac{3}{9}\right)$
$+ \left(\frac{4}{10} \times \frac{3}{9} \times \frac{6}{8}\right) = \frac{121}{450} \approx 0,27$

2.2.3 P(at least 1 blue marble)
$= 1 - P(\text{no blue marble})$
$= 1 - P(WWW)$
$= 1 - \left(\frac{6}{10} \times \frac{6}{10} \times \frac{6}{10}\right)$
$= \frac{98}{125}$
$= 0,784$

Worksheet 11 B

3.



P(Lightning win)
$= P(\text{Dry/Win}) + P(\text{Wet/Win})$
$= (0,65 \times 0,93) + (0,35 \times 0,31)$
$= 0,6045 + 0,1085 = 0,713$

Worksheet 11 D

2.6 P(Brown hair) = $\frac{700}{1861}$

P(Brown eyes) = $\frac{636}{1861}$

P(Brown hair and brown eyes) = $\frac{327}{1861}$
= 0,1757...

P(Brown hair) x P(Brown eyes)
= $\frac{700}{1861} \times \frac{636}{1861}$ = 0,12854...

P(Brown hair) x P(Brown eyes)
≠ P(Brown hair and brown eyes)

∴ Not independent

3.1 e = 250 f = 1000

a = 125 c = 625

b = 250 d = 375

	< 5	> 5	Total
28 years +	125	125	250
< 28 years	500	250	750
Total	625	375	1000

3.2 P(older than 28 years)= $\frac{250}{1000}$

P(less than 5) = $\frac{625}{1000}$

$\frac{250}{1000} \times \frac{625}{1000}$ = 0,15625

P(older than 28 years and less than 5)
= $\frac{125}{1000}$ = 0,125

Because P(older than 28 and < 5)
≠ P(28+) x P(<5)
the number of accidents made by a person is not independent of the person's age.

Worksheet 11 E

1.1 3 x 5 x 2 = 30

1.2 P(Blue shirt) = $\frac{1 \times 3 \times 2}{30} = \frac{1}{5}$

2.1 1 x 4 x 7 x 5 x 2
sherry starter main pud. tea or coffee

= 280

2.2 1 x 3 x 5 x 5 x 2 = 150

3. 5! = 120

4. 8! = 40 320

5. (6 – 1)! = 5! = 120

6.1 8 books ∴ 8! = 40 320

6.2 3! x 5! x 2! = 1440

7.1 12! = 479 001 600

7.2 1 x 10! x 1 = 3 628 800

7.3 9, 10 and 11 form 1 unit
∴ 1 unit + 9 other
= 10 units
∴ 10! x 3! = 21 772 800

9, 10 and 11 can be arranged in any order.

Worksheet 11 F

1.1 9 x 9 = 81

1.2 9 x 8 x 7 x 6 or $({}^{9}P_{4})$ = 3024

1.3 1 x 9 x 9 x 9 = 729

1.4 1 x 7 x 6 x 1 = 42

1.5 Total number of possible outcomes:
9^4 = 6561 (may be repeated)

Number of possible outcomes without a 5 :
8^4 = 4096 (may be repeated)

∴ number with at least one 5:
6561 – 4096 = 2465

Worksheet 11 F

2. □ □ □ 26 letters
10 numbers
∴ 26 x 10 x 9 = 2340
(may not be repeated)

3.1 6 numbers
6 x 6 x 6 x 6 x 6 = 6^5 = 7776
(may be repeated)

3.2 6 x 5 x 4 x 3 x 2 = 720 or $({}^{6}P_{5})$
(may not be repeated)

3.3 4,5 6 5

□ □ □

2 x 6 x 1 = 12

4. 0 1 2 3 4 5

□ □ □ □

1,2,3,4,5 ↑ ↑ 0,2,4 ↑

5 x 6 x 6 x 3

5 x 6 x 6 x 3 = 540

5. Letters: A – L : 12
numbers: 0 – 9 : 10
Symbols: 2

5.1 12 x 11 x 10 x 10 x 9 x 8 x 2

= 1 900 800

5.2 12 x 11 x 10 x 10^3 x 2

= 2 640 000

5.3 ABC: 1 x 1 x 1 x 10 x 9 x 8 x 2

= 1440 options

∴P(begins with ABC) = $\frac{1440}{1\,900\,800}$
= 0,000$\dot{7}\dot{5}$

Worksheet 11 F

5.4 n(one 2)
= 12 x 11 x 10 x 3 x (1 x 9 x 8) x 2
= 570 240

$$P(\text{one 2}) = \frac{570\,240}{1\,900\,800}$$
= 0,3 (no repetition allowed)

5.5 n (6 characters)
= 12 x 11 x 10 x 10 x 9 x 2
= 237 600

n (7 characters)
= 12 x 11 x 10 x 10 x 9 x 8 x 2
= 1 900 800

n (8 characters)
= 12 x 11 x 10 x 10 x 9 x 8 x 7 x 2
= 13 305 600

P(6 characters)

$$= \frac{237\,600}{237\,600 + 1\,900\,800 + 13\,305\,600}$$
= 0,01538 ≈ 0,015

5.6 P(7 characters)
$$= \frac{24^7}{24^6 + 24^7 + 24^8}$$
= 0,04

Worksheet 11 G

1.1 $\frac{8!}{2!}$ = 20 160 (8 letters, 2 repeated)

1.2 $\frac{1 \times 7!}{2!}$ = 2520

Worksheet 11 G

1.3 $\frac{1 \times 6! \times 1}{2!}$ = 360

1.4 P(begins with G & ends with Y)
$$= \frac{360}{20\,160} = 0{,}01785...$$
≈ 0,02

2.1 $\frac{8!}{3!}$ = 6720 (8 letters, 3 repeated)

2.2.1 n(begins with D)= 1 x $\frac{7!}{3!}$ = 840

1 – D begin
7! – 7 letters left
3! – 3A^s repeated

∴ P(word begins with a D) = $\frac{840}{6720} = \frac{1}{8}$

2.2.2 n (begins with an A) $= \frac{1 \times 7!}{2!}$
= 2520

∴ P(begins with an A)= $\frac{2520}{6720} = \frac{3}{8}$

2.2.3 n (Begin and end with A)

A _ _ _ _ _ _ **A**
1 x 6! x 1 =720

6 letters left, none repeated

∴P(begins and ends with an A)

$$= \frac{720}{6720}$$
$$= \frac{1}{16}$$
= 0,0625

Worksheet 11 G

3.1 6! = 720

3.2 0,05 x 720 = 36

4. Possible combinations with "letters" from MISSISIPPI:

$$\frac{10!}{3!\ 2!\ 4!}$$ (10 letters, 3 S^s, 4 I^s, 2 P^s)
= 12 600

Number that begins and ends with an **S**:

S_ _ _ _ _ _ _ _**S**

$$1 \times \frac{8!}{4!\ 2!} \times 1 = 840$$
(I) (P)

Number that begins and ends with a **P**:

P_ _ _ _ _ _ _ _**P**

$$1 \times \frac{8!}{4!\ 3!} \times 1 = 280$$
(I) (S)

Number that begins and ends with an **I**:
I_ _ _ _ _ _ _ _**I**

$$1 \times \frac{8!}{2!\ 3!\ 2!} \times 1 = 1680$$
(I)(S)(P)

∴ **Total number that begins and ends with the same letter:**

840 + 280 + 1680 = 2800

∴P(**begins and ends with the same letter)**

$$= \frac{2800}{12\,600}$$
$$= \frac{2}{9}$$
= 0,22

Worksheet 11 G

5. 7 numbers ∴ $\frac{7!}{4!\ 3!}$ = 35

∴ first guess correct
$$= \frac{1}{35} = 0{,}02857... \approx 0{,}03$$
(divide by 3! because the order of the 3 numbers does not matter.)

6.1 49 x 48 x 47 x 46 x 45 x 44
= 1,00683 x 10^{10} = $\frac{49!}{43!}$ or $^{49}P_6$

6.2 $\frac{49}{43!\ 6!}$ = 13 983 816
(divide by 6! because the order of the 6 numbers does not matter.)

6.3 $\frac{1}{13\,983\,816}$

= 0,000 000 071 ≈ 0

6.4 13 983 816 x R3,50

= R48 943 356

(nearly 49 million rand!)